LECTURE 1 REVIEW:

Exercises on Data Scales, Research Design, Variables and Problem Statements

LECTURE 2

Introduction to Probability

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Purpose of Lecture

1. To introduce the concept of "P"
2. To lay the foundation for probability references used in descriptive and inferential statistics
3. This is an overview and is limited to introductory probability

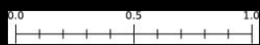
Introduction to Probability

Definition:

1. Probability or "P" represents the likelihood that an event can or will occur.

2. *Occurs on a Continuous Scale From 0 to 1*

0 = Event Will Not Occur



.5 = Two Events Equal Chance To Occur

1 = Event Will Occur

3. Points between 0 and 1 always reported as a decimal, never as a fraction.

Terminology

Randomness – All have equal chance of being selected

Independence – Selection of one does not influence the selection of others

Mutually Exclusive or Mutual Exclusivity – Two events cannot occur at the same time.

Discrete Random Variable – where the variable may take on only a countable number of distinct values (e.g. # patients in patient pool)

Sampling With Replacement – Sample, replace sample, then re-sample. Overall number in population (TO) stays the same for each sample.

Sampling Without Replacement – Sample, reduce TO, Resample. Population reduces by size of sample with each re-sample.

Teaching Approach

Single Event Probabilities: Where only a single event leads to a single.

Multiple Event Probability: Where single or several events lead to one or more outcomes.

Multiple Events Taught as Addition or Multiplication Rule.

Single Event Probabilities

Simple, Single Event Probabilities

$$P(E) = \frac{\#Favorable\ Outcomes}{\#Possible\ Outcomes} = \frac{E}{TO}$$

$$P(_{Female}) = \frac{10}{17} = .588$$

Question: What is the probability that a female will be drawn on a single random draw from the population above?

Interpretation: Female Likely to be selected 58.8% of the time

Reasoning

Event Occurring: $P(E_1)=(E/TO)$

$$P(Female) = \frac{10}{17} = .588$$

Event Not Occurring: $P(E_0) = 1 - P(E_1)$

$$P \neq (Female) = 1 - .588 = .412$$

Not Occurring + Occurring: $P(E_1) + P(E_0) = 1$

$$P(Female\ .588) + P \neq (Female\ .412) = 1$$

Simple Exercises in Single Event Probabilities

Two freshmen in a class of 91 students failed during the semester. What is the probability that a freshman will fail? Ans: 2/91= .02198

A dental student doctor at a health fair found the following treatment needs:

Ward	# Needing Treatment	Total Screened
Ward 1	35	268
Ward 2	62	145
Ward 3	45	320
Ward 4	26	64
Total	168	797

1. What is the probability that a randomly selected patient will need treatment?
2. What is the probability that a patient from Ward 1 will need to be seen?
3. What is the probability that a patient from Ward 4 will not need further care?

Answers:

1. 168/797 = .2108
2. 35/268 = .1306
3. 1 – (26/64) = .5938

Multiple Event Probabilities

Multiple Events Probability

Let's consider as 2 rules:

- Addition Rule – "Either" "Or" **(Add separate probabilities)**
  - Mutually Exclusive Events
  - Non-Mutually Exclusive Events
- Multiplication Rule – "And" **(Multiply separate probabilities)**
  - Independent Events
  - Dependent (Conditional) Events

Mutual Exclusivity

Addition Rule: Occurrence of two or more mutually exclusive events equals the sum of their separate probabilities. The "Or" and "Either" Rule.

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

Probability of a 6 "or" a 3 on a single throw of a die:

P(E_6 and E_3) = 0

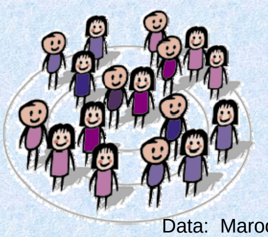
Separate probabilities:

P(6) = 1/6 = .1667   P(3) = 1/6 = .1667

P($_6$ or $_3$) = P(6) + P(3) = 1/6+1/6 = 2/6 = 1/3 = .3333

Therefore, we expect to get a 6 or a 3 on 1 of 3 tosses of a fair die, or 33.33% of the time.

Mutual Exclusivity



$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

What is the probability of picking purple **"or"** pink clothing on a single selection?

Data: Maroon = 3, Pink = 6, Purple = 8, n=17

$$P = \left(Purple\frac{8}{17} + Pink\frac{6}{17}\right) = (.4705 + .3529) = .824$$

Mutual Exclusivity

Addition Rule: Occurrence of two or more mutually exclusive events equals the sum of their separate probabilities. The "Or" and "Either" Rule.

2. Probability of drawing "Either" an Ace "Or" a Ten on a single draw of cards:

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

P(ACE) = 4/52 = 1/13 = .0769   P(Ten) = 4/52 = 1/13 = .0769

P(ACE or TEN) = P(ACE) + P(TEN) = 1/13+1/13 = 2/13 = .1538

Therefore, we expect to get an ACE or a TEN on 2 of 13 draws from a deck of cards, or 15.38% of the time.

Non-Mutual Exclusivity

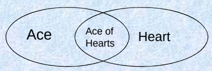
Addition Rule For Non-Mutually Exclusive Events

- **Probabilities are double counted, since both probabilities can occur**
- **Adjust by subtracting product of both probabilities**

Equation: $P(E_1) + P(E_2) - P(E_1E_2)$

What is the probability of drawing an ACE "or" a Heart on a single draw

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$



Ace of Hearts = 1/52=.01923

P(Ace 4/52) + P(Heart 13/52) – (P4/52*13/52) = .3269 - .0192 = .3077

Without Adjusting for Double Counting: P = .3269

Non-Mutual Exclusivity

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

What is the probability of picking purple clothing **"or"** a female on a single selection?

Data: Female = 10, Pink = 6, Purple = 8, n=17

$$P = \left(Purple\,\frac{8}{17} + Female\,\frac{10}{17}\right) - \left(Purple\,\frac{8}{17} * Female\,\frac{10}{17}\right)$$

$$= (.4706 + .5882) - (.4706 * .5882)$$

$$= 1.058 - .2768 = .7812$$

Non-Mutual Exclusivity

Class Problem

All Europeans who descended from Black Plague survivors in Smallville, Notia were screened for the HIV resistance CCR5-delta32 mutation. The results are below:

HIV Resistance		Male	Female	Total
	Yes	46	38	84
	No	21	26	47
	Total	67	64	131

What is the probability that a randomly subject would be HIV resistant or female?

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$P = \left(\text{Resistant}\ \frac{84}{131} + \text{Female}\ \frac{64}{131}\right) - \left(\text{Resistant}\ \frac{84}{131} * \text{Female}\ \frac{64}{131}\right)$$

$$= (.6412 + .4885) - (.6412 * .4885)$$

$$= 1.1297 - .3132 = .8165$$

Multiplication Rule

Multiplication Rule for Multiple Events

- Multiple probabilities joined with "And"
- Equals the Product of Separate Probabilities

Independent Events: $P(E_1E_2) = P(E_1)*P(E_2)$

What is the probability of throwing a 1 and a 4 on a single roll of dice?

Ans: $P(E_1 1/6)*P(E_2 1/6) = 1/36 = .0278$

Multiple Events

Independent Events: $P(E_1E_2) = P(E_1)*P(E_2)$

What is the probability of picking purple clothing **"and"** a female on a single selection?

Data: Female = 10, Pink = 6, Purple = 8, n=17

$$P = \left(Purple\ \frac{8}{17} * Female\ \frac{4}{17} \right) = (.4706 * .235) = .111$$

Note: Purple Female = 4/17 = .235

Non-Mutual Exclusivity

Class Problem

All Europeans who descended from Black Plague survivors in Smallville, Notia were screened for the HIV resistance CCR5-delta32 mutation. The results are below:

HIV Resistance		Male	Female	Total
	Yes	46	38	84
	No	21	26	47
	Total	67	64	131

What is the probability that a subject would be HIV resistant and female?

$$P = \left(\text{Resistant} \quad \frac{84}{131} * \text{Resistant} \quad \text{Female} \quad \frac{38}{64} \right)$$

$= (.6412 \quad * .5938)$

$= .3807$

Multiplication Rule for Multiple Events

Dependent (Conditional) Events

- 2 Events occur in specific order
- P(B|A) = Probability of "B" occurring, given that "A" has occurred.
- Conditional in that "A" is required before "B" occurs
- Equation: $P(B|A) = \frac{P(B \text{ and } A)}{P(A)}$
- Condition is Independent when P(B) = P(B|A)

Multiple Events

Independent Events: $P(E_1E_2) = P(E_1)*P(E_2)$

What is the probability of picking purple clothing given that a female was picked first?

Data: Female = 10, Pink = 6, Purple = 8, n=17

Independent because P(B)= P(B|A)

$$P(B \mid A) = P(\text{Purple} \mid \text{Female}) = \frac{P(B \text{ and } A)}{P(A)}$$

$$= \frac{P(\frac{8}{17} * \frac{10}{17})}{P(\frac{10}{17})} = \frac{.2768}{.5882} = .4705$$

Multiplication Rule for Multiple Events
Dependent (Conditional) Events

What is Probability of "Moderate" political view, given that the subject is "Female"?

•Easiest solution: Cross-Tabulate and reduce to simple probability

		Political views			
		Liberal	Moderate	Conservative	Total
Gender	Male	17	29	14	60
	Female	30	24	23	77
	Total	47	53	37	137

Simple Answer: P(Moderate - Female) = 24/77 = .311
Now Calculate using Equation:
P(B|A)=(Moderate and Female)/Female=(24/137)/(77/137)=.312